Specialist Mathematics

Unit 3– Problem-solving and modelling task

Student name: Yiran (Canace) Chen

Student number: s45587

Date handed out: 31/01/2020

Checkpoint dates: 7/02/2020, 14/02/2020, 21/02/2020

Due date: 28/02/2020

1 Introduction

This problem-solving and modelling task is to generate a piece of melody using a suitable mathematical method. To achieve this purpose, an appropriate sample of music was used as reference data to develop the model. Both technological, for example, Excel and graphic calculator, and mathematical procedures including matrices were used to generate the melody. A trail of simulated notes was included in this report to help with the further evaluation and analysis on comparing the melody generated by mathematics procedure and the melody composed by an artist.

2 Considerations

2.1 Observations

1. According to the sample music, it was observed that there are 17 types of notes including E4, F4, G4, A4, B4, C5 E5, F5, G5, A5, B5, C6, D6, E6, F6, G6, A6 and there are 415 notes used in the melody in total.

2. Since melody shows a long-term behaviour and transits in various notes, the Markov chain matrix was chosen to generate the melody.

2.2 Assumptions

1. Since the purpose was to generate a piece of melody using Markov Chain matrix, all the other factors such as length of each note, sharps and flats were not considered in the generated melody. In other words, each note was assumed to be of equal time and there was assumed to be no sharps or flats.

2. According to the sample music, it is assumed that the melody starts with the note A4.

3. According to the rule of Markov Chain matrix, it is assumed that the next note is randomly selected based on the allocated probability.

4. According to the rule of Markov Chain matrix, there should always be a note after another. Therefore, the note after the last note of the sample music was assumed to be the first note of the sample music.

5. Based on Observation 1, it was assumed to have 50 simulated notes in the trail.

3 Mathematical concepts and procedures

To generate the melody, a sample music was needed as reference data and to demonstrate the application of matrices. The Markov Chain matrix was chosen to be the basis of the mathematical model since it makes various transition in states of the system and shows long-term behaviour. To apply the Markov Chain matrix, the probability of each note selected after the previous note was needed. A table was created to collect all the data that was needed to in order to apply the Markov Chain

matrix. By this way, it is clearer to see how each data affected the Markov chain matrix and also made it easier to adjust the notes and probability allocations for development purpose.

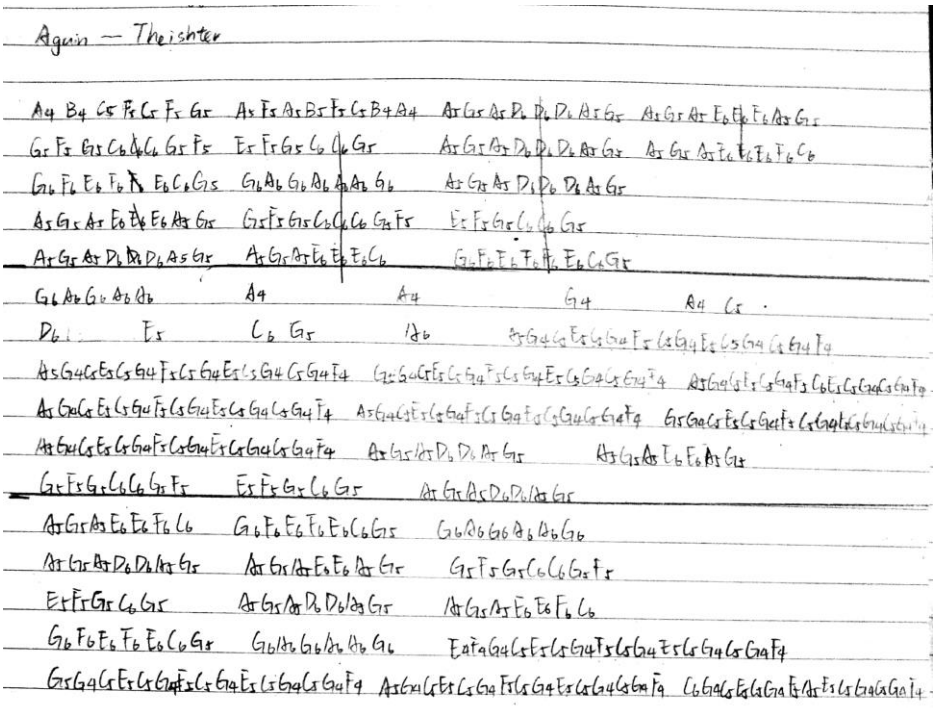
For further investigation, a trial of simulated notes was included in the report to helped to compare with the sample music.

4 Determining the solution and refinement of the models

4.1 Determining the transition matrix

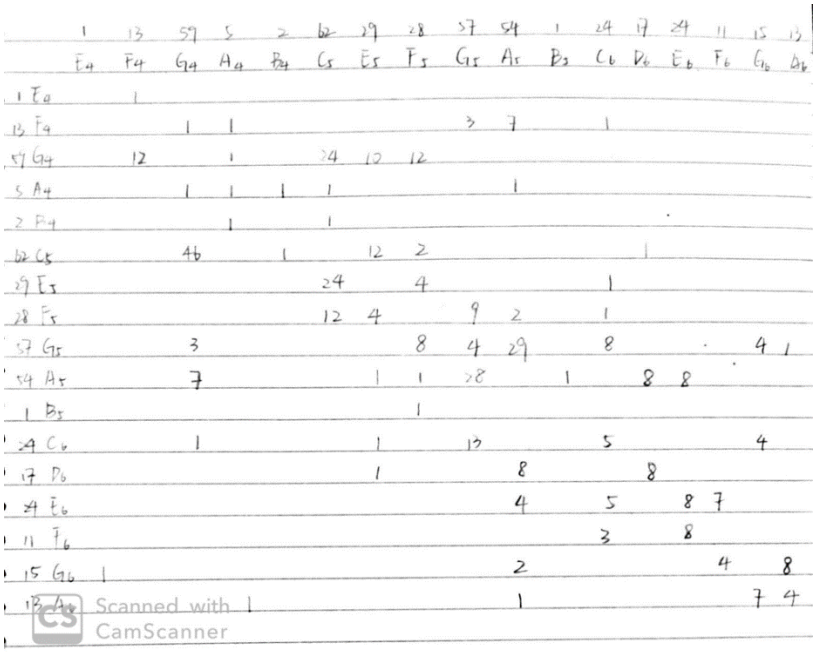
Firstly, a sample music was found as reference data to help to develop the melody (see appendix). It was chosen because: a) it has a variety of different types of notes which will make the music sound more interesting; b) it has a few repetitions which will be less time consuming when determining the transition matrix; c) instead of notes with different lengths, most of the notes in the sample music were eighth notes which means the generated melody will be more likely to sound similar to the sample music; d) it only contains sharps in the sample music.

The sheet music of the sample was converted to alphabet version (work shown in Figure 1).



(Figure 1: A photograph showing the hand work of the process of converting the sheet music into alphabet version.)

As the Markov Chain matrix calculates the probability associated with the state of a system as it makes various transitions, the note that is after one another as well as its frequency in this case was determined and the data was collected in the table below.



(Table 1: The table shows the number of times each note is after the previous note.)

Based on Table 1, a table of the transition matrix of the Markov Chain matrix was created. The sum of probabilities in the transition matrix was checked to be equal to 1.

The Transition Matrix

	E4	F4	G4	A4	B4	C5	E5	F5	G5	A5	B5	C6	D6	E6	F6	G6	A6
E4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/15	0
F4	1	0	12/59	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G4	0	1/13	0	1/5	0	23/31	0	0	1/19	7/54	0	1/24	0	0	0	0	0
A4	0	1/13	1/59	1/5	1/2	0	0	0	0	0	0	0	0	0	0	0	1/13
B4	0	0	0	1/5	0	1/62	0	0	0	0	0	0	0	0	0	0	0
C5	0	0	24/59	1/5	1/2	0	24/29	3/7	0	0	0	0	0	0	0	0	0
E5	0	0	10/59	0	0	6/31	0	1/7	0	1/54	0	1/24	1/17	0	0	0	0
F5	0	0	12/59	0	0	1/31	4/29	0	8/57	1/54	1	0	0	0	0	0	0
G5	0	3/13	0	0	0	0	0	9/28	4/57	14/27	0	13/24	0	0	0	0	0
A5	0	7/13	0	1/5	0	0	0	1/14	29/57	0	0	0	8/17	1/6	0	2/15	1/13
B5	0	0	0	0	0	0	0	0	0	1/54	0	0	0	0	0	0	0
C6	0	1/13	0	0	0	0	1/29	1/28	8/57	0	0	5/24	0	5/24	3/11	0	0
D6	0	0	0	0	0	1/62	0	0	0	4/27	0	0	8/17	0	0	0	0
E6	0	0	0	0	0	0	0	0	0	4/27	0	0	0	1/3	8/11	0	0
F6	0	0	0	0	0	0	0	0	0	0	0	0	0	7/24	0	4/15	0
G6	0	0	0	0	0	0	0	0	4/57	0	0	1/6	0	0	0	0	7/13
A6	0	0	0	0	0	0	0	0	1/57	0	0	0	0	0	0	8/15	4/13
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

(Table 2: The table shows the transition matrix of the Markov Chain matrix.)

	E4	F4	G4	A4	B4	C5
E4	0	0	0	0	0	0
F4	1	0	0.203389830508475	0	0	0
G4	0	0.076923076923076	0	0.2	0	0.741935483870968
A4	0	0.076923076923076	0.0169491525423729	0.2	0.5	0
B4	0	0	0	0.2	0	0.016129032258064
C5	0	0	0.406779661016949	0.2	0.5	0
E5	0	0	0.169491525423729	0	0	0.193548387096774
F5	0	0	0.203389830508475	0	0	0.032258064516129
G5	0	0.230769230769231	0	0	0	0
A5	0	0.538461538461538	0	0.2	0	0
B5	0	0	0	0	0	0
C6	0	0.076923076923076	0	0	0	0
D6	0	0	0	0	0	0.016129032258064
E6	0	0	0	0	0	0
F6	0	0	0	0	0	0
G6	0	0	0	0	0	0
A6	0	0	0	0	0	0
	=SUM(B2:B18)	=SUM(C2:C18)	=SUM(D2:D18)	=SUM(E2:E18)	=SUM(F2:F18)	=SUM(G2:G18)

(Table 3: The table shows part of the calculation formulas for Table 2.)

To generate a piece of melody, an initial state was needed. From Assumption 2, the first note is A4, therefore, the probability of A4 is 1 and the probabilities for the other notes are 0. State 1 was calculated by multiplying the transition matrix with the initial state. The second note was determined by generating random value based on the probabilities using Excel (table shown in figure 2).

E4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/15	0	0
F4	1	0	12/59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G4	0	1/13	0	1/5	0	23/31	0	0	1/19	7/54	0	1/24	0	0	0	0	0	0
A4	0	1/13	1/59	1/5	1/2	0	0	0	0	0	0	0	0	0	0	0	1/13	1
B4	0	0	0	1/5	0	1/62	0	0	0	0	0	0	0	0	0	0	0	0
C5	0	0	24/59	1/5	1/2	0	24/29	3/7	0	0	0	0	0	0	0	0	0	0
E5	0	0	10/59	0	0	6/31	0	1/7	0	1/54	0	1/24	1/17	0	0	0	0	0
F5	0	0	12/59	0	0	1/31	4/29	0	8/57	1/54	1	0	0	0	0	0	0	0
G5	0	3/13	0	0	0	0	0	3/28	4/57	14/27	0	13/24	0	0	0	0	0	0
A5	0	7/13	0	1/5	0	0	0	1/14	23/57	0	0	0	8/17	1/6	0	2/15	1/13	0
B5	0	0	0	0	0	0	0	0	0	1/54	0	0	0	0	0	0	0	0
C6	0	1/13	0	0	0	0	1/29	1/28	8/57	0	0	5/24	0	5/24	3/11	0	0	0
D6	0	0	0	0	0	1/62	0	0	0	4/27	0	0	8/17	0	0	0	0	0
E6	0	0	0	0	0	0	0	0	0	4/27	0	0	0	1/3	8/11	0	0	0
F6	0	0	0	0	0	0	0	0	0	0	0	0	0	7/24	0	4/15	0	0
G6	0	0	0	0	0	0	0	0	4/57	0	0	1/6	0	0	0	0	7/13	0
A6	0	0	0	0	0	0	0	0	1/57	0	0	0	0	0	0	8/15	4/13	0
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

1	
0	0
0	0
0.2	0.2
0.2	0.4
0.2	0.6
0.2	0.8
0	0.8
0	0.8
0	0.8
0.2	1
0	1
0	1
0	1
0	1
0	1
0	1
0	1
C5	
B4	

(Figure 2: Screenshot of the process of determining the second note.)

1	
=MMULT(B2:R18,T2:T18)	=SUM($A$22:A22)
	=SUM($A$22:A23)
	=SUM($A$22:A24)
	=SUM($A$22:A25)
	=SUM($A$22:A26)
	=SUM($A$22:A27)
	=SUM($A$22:A28)
	=SUM($A$22:A29)
	=SUM($A$22:A30)
	=SUM($A$22:A31)
	=SUM($A$22:A32)
	=SUM($A$22:A33)
	=SUM($A$22:A34)
	=SUM($A$22:A35)
	=SUM($A$22:A36)
	=SUM($A$22:A37)
	=SUM($A$22:A38)
=INDEX(A$2:A$18,COUNTIF(B$22:B$38,"<="&RAND())+1)	
B4	

(Table 4: The table shows the calculation formulas used to determine the second note.)

Similarly, the third note was determined by multiplying the transition matrix with State 1. Then the steps used to determine the second note were repeated.

Determination of State 2

2	
0	0
0.04067797	0.040678
0.21431302	0.254991
0.14338983	0.398381
0.04322581	0.441607
0.22135593	0.662963
0.07631169	0.739274
0.05083328	0.790108
0.1037037	0.893811
0.04	0.933811
0.0037037	0.937515
0	0.937515
0.03285544	0.97037
0.02962963	1
0	1
0	1
0	1
G4	
E6	

(Table 5: The table shows the process of determining the third note.)

2	
=MMULT(B2:R18,A22:A38)	=SUM($D$22:D22)
	=SUM($D$22:D23)
	=SUM($D$22:D24)
	=SUM($D$22:D25)
	=SUM($D$22:D26)
	=SUM($D$22:D27)
	=SUM($D$22:D28)
	=SUM($D$22:D29)
	=SUM($D$22:D30)
	=SUM($D$22:D31)
	=SUM($D$22:D32)
	=SUM($D$22:D33)
	=SUM($D$22:D34)
	=SUM($D$22:D35)
	=SUM($D$22:D36)
	=SUM($D$22:D37)
	=SUM($D$22:D38)
=INDEX(A$2:A$18,COUNTIF(E$22:E$38,"<="&RAND())+1)	
E6	

(Table 6: The table shows the calculation formulas used to determine the second note.)

A mini trial of 30 simulated notes was then generated based on the steps which determined the first three notes. A spreadsheet of the mini trail including states and the process of finding each note was created to help with further refinement.

0	0.8		0.076312	0.7393		0.089103	0.651084		0.094549	0.606639		0.091526	0.5619		0.0896	0.5346		0.0858	0.5092		0.0834	0.4917
0	0.8		0.050833	0.7301		0.080255	0.731338		0.072144	0.678784		0.081755	0.6436		0.076	0.6106		0.0776	0.5869		0.0745	0.5662
0	0.8		0.103704	0.8938		0.053745	0.785083		0.121004	0.799787		0.100023	0.7436		0.1263	0.7369		0.1191	0.706		0.1293	0.6955
0.2	1		0.04	0.9338		0.127374	0.912457		0.083447	0.883234		0.1191	0.8627		0.1056	0.8426		0.1206	0.8265		0.1166	0.8122
0	1		0.003704	0.9375		0.000741	0.913197		0.002359	0.885592		0.001545	0.8643		0.0022	0.8448		0.002	0.8285		0.0022	0.8144
0	1		0	0.9375		0.028304	0.941501		0.028381	0.913973		0.04009	0.9044		0.0404	0.8852		0.0456	0.8741		0.0466	0.861
0	1		0.032855	0.9704		0.024958	0.966459		0.034202	0.948175		0.032002	0.9364		0.036	0.9212		0.0358	0.9099		0.0377	0.8987
0	1		0.02963	1		0.015802	0.982261		0.030423	0.978598		0.027267	0.9637		0.035	0.9562		0.0362	0.9461		0.0409	0.9397
0	1		0	1		0.008642	0.990903		0.00655	0.985147		0.011398	0.9751		0.0123	0.9685		0.0151	0.9613		0.0166	0.9563
0	1		0	1		0.007277	0.998181		0.009469	0.994616		0.016121	0.9912		0.0185	0.9869		0.0226	0.9839		0.0246	0.9809
0	1		0	1		0.001819	1		0.005384	1		0.008829	1		0.0131	1		0.0161	1		0.0191	1
G4			G5			C5			E5			G4			G5			G4			A5	
B4			G4			A4			A5			E5			F4			A5			C5	
11			12			13			14			15			16			17			18	
0.0019	0.0019		0.002009	0.002		0.002078	0.002078		0.002143	0.002143		0.00219	0.0022		0.0022	0.0022		0.0023	0.0023		0.0023	0.0023
0.0344	0.0363		0.033801	0.0358		0.033349	0.035427		0.032974	0.035117		0.032673	0.0349		0.0324	0.0347		0.0322	0.0345		0.0321	0.0343
0.1568	0.1931		0.154085	0.1899		0.151906	0.187333		0.150105	0.185222		0.14865	0.1835		0.1475	0.1821		0.1465	0.181		0.1457	0.18
0.0124	0.2055		0.012311	0.2022		0.012256	0.199589		0.012215	0.197438		0.012184	0.1957		0.0122	0.1943		0.0121	0.1931		0.0121	0.1922
0.0053	0.2107		0.005167	0.2074		0.005101	0.20469		0.005047	0.202485		0.005005	0.2007		0.005	0.1992		0.0049	0.1981		0.0049	0.1971
0.1667	0.3774		0.163623	0.371		0.160937	0.365627		0.158853	0.361338		0.157089	0.3578		0.1557	0.3549		0.1545	0.3526		0.1536	0.3507
0.0772	0.4546		0.075906	0.4469		0.074768	0.440395		0.073888	0.435226		0.07314	0.4309		0.0725	0.4275		0.0721	0.4246		0.0717	0.4223
0.072	0.5266		0.070919	0.5178		0.070424	0.51082		0.069799	0.505026		0.069422	0.5004		0.069	0.4965		0.0688	0.4934		0.0685	0.4908
0.1312	0.6578		0.133251	0.6511		0.133486	0.644306		0.134516	0.639541		0.134843	0.6352		0.1354	0.6319		0.1357	0.6291		0.136	0.6269
0.1251	0.7829		0.12522	0.7763		0.126634	0.770939		0.126987	0.766529		0.12775	0.7629		0.1281	0.76		0.1285	0.7576		0.1288	0.7557
0.0023	0.7852		0.002316	0.7786		0.002319	0.773258		0.002345	0.768874		0.002352	0.7653		0.0024	0.7624		0.0024	0.76		0.0024	0.758
0.0519	0.8371		0.05282	0.8314		0.053823	0.827082		0.054487	0.82336		0.055132	0.8204		0.0556	0.818		0.056	0.816		0.0563	0.8144
0.0392	0.8763		0.039651	0.8711		0.03985	0.866931		0.040109	0.863469		0.04025	0.8607		0.0404	0.8584		0.0405	0.8565		0.0406	0.855
0.0478	0.9241		0.049779	0.9209		0.051134	0.918065		0.05241	0.91588		0.053364	0.914		0.0542	0.9126		0.0549	0.9114		0.0554	0.9104
0.021	0.9451		0.021986	0.9428		0.022832	0.940897		0.023486	0.939365		0.024047	0.9381		0.0245	0.9371		0.0249	0.9362		0.0252	0.9356
0.0301	0.9753		0.031175	0.974		0.032144	0.973041		0.032854	0.97222		0.033479	0.9716		0.034	0.971		0.0344	0.9706		0.0347	0.9702
0.0247	1		0.025981	1		0.026959	1		0.02778	1		0.02843	1		0.029	1		0.0294	1		0.0298	1
G5			G4			F6			D6			F5			A6			A4			C5	

(Figure 3: A screenshot of part of the mini trail.)

	=SUM($A$22:A29)			=SUM($D$22:D29)			=SUM($G$22:G29)	
	=SUM($A$22:A30)			=SUM($D$22:D30)			=SUM($G$22:G30)	
	=SUM($A$22:A31)			=SUM($D$22:D31)			=SUM($G$22:G31)	
	=SUM($A$22:A32)			=SUM($D$22:D32)			=SUM($G$22:G32)	
	=SUM($A$22:A33)			=SUM($D$22:D33)			=SUM($G$22:G33)	
	=SUM($A$22:A34)			=SUM($D$22:D34)			=SUM($G$22:G34)	
	=SUM($A$22:A35)			=SUM($D$22:D35)			=SUM($G$22:G35)	
	=SUM($A$22:A36)			=SUM($D$22:D36)			=SUM($G$22:G36)	
	=SUM($A$22:A37)			=SUM($D$22:D37)			=SUM($G$22:G37)	
	=SUM($A$22:A38)			=SUM($D$22:D38)			=SUM($G$22:G38)	
=INDEX(A$2:A$18,COUNTIF(B$22:B$3			=INDEX(A$2:A$18,COUNTIF(E$22:E$38,"<=			=INDEX(A$2:A$18,COUNTIF(H$22:H$38,"<="&R		
B4			G4			A4		
11			12			13		
=MMULT(B2:R18,A	=SUM($A$43:A43)		=MMULT(B2:R18,A43:A	=SUM($D$43:D43)		=MMULT(B2:R18,D43:D	=SUM($G$43:G43)	
	=SUM($A$43:A44)			=SUM($D$43:D44)			=SUM($G$43:G44)	
	=SUM($A$43:A45)			=SUM($D$43:D45)			=SUM($G$43:G45)	
	=SUM($A$43:A46)			=SUM($D$43:D46)			=SUM($G$43:G46)	
	=SUM($A$43:A47)			=SUM($D$43:D47)			=SUM($G$43:G47)	
	=SUM($A$43:A48)			=SUM($D$43:D48)			=SUM($G$43:G48)	
	=SUM($A$43:A49)			=SUM($D$43:D49)			=SUM($G$43:G49)	
	=SUM($A$43:A50)			=SUM($D$43:D50)			=SUM($G$43:G50)	
	=SUM($A$43:A51)			=SUM($D$43:D51)			=SUM($G$43:G51)	
	=SUM($A$43:A52)			=SUM($D$43:D52)			=SUM($G$43:G52)	
	=SUM($A$43:A53)			=SUM($D$43:D53)			=SUM($G$43:G53)	
	=SUM($A$43:A54)			=SUM($D$43:D54)			=SUM($G$43:G54)	
	=SUM($A$43:A55)			=SUM($D$43:D55)			=SUM($G$43:G55)	
	=SUM($A$43:A56)			=SUM($D$43:D56)			=SUM($G$43:G56)	
	=SUM($A$43:A57)			=SUM($D$43:D57)			=SUM($G$43:G57)	
	=SUM($A$43:A58)			=SUM($D$43:D58)			=SUM($G$43:G58)	
	=SUM($A$43:A59)			=SUM($D$43:D59)			=SUM($G$43:G59)	
=INDEX(A$2:A$18,COUNTIF(B$43:B$5			=INDEX(A$2:A$18,COUNTIF(E$43:E$59,"<=			=INDEX(A$2:A$18,COUNTIF(H$43:H$59,"<="&R		
A5			G5			c5		

(Figure 4: The corresponding spreadsheet of formulas used to generate the mini trail.)

According to the spreadsheet, it shows that the probabilities of each note stay the same from the 27th note. Therefore State 26 is the steady state or long-term behaviour of the melody (evidence shown in Figure 5).

26		27		28		29	
0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024
0.0315	0.0339	0.0314	0.0338	0.0314	0.0338	0.0314	0.0338
0.1429	0.1767	0.1427	0.1766	0.1426	0.1764	0.1425	0.1763
0.0121	0.1888	0.0121	0.1886	0.0121	0.1885	0.0121	0.1884
0.0048	0.1936	0.0048	0.1935	0.0048	0.1933	0.0048	0.1932
0.1502	0.3438	0.1501	0.3435	0.1499	0.3433	0.1498	0.3431
0.0702	0.4141	0.0702	0.4137	0.0701	0.4134	0.0701	0.4131
0.0677	0.4817	0.0676	0.4813	0.0676	0.481	0.0676	0.4807
0.1371	0.6188	0.1371	0.6185	0.1372	0.6182	0.1372	0.6179
0.1299	0.7487	0.1299	0.7484	0.1299	0.7481	0.13	0.7479
0.0024	0.7511	0.0024	0.7508	0.0024	0.7505	0.0024	0.7503
0.0575	0.8086	0.0576	0.8084	0.0576	0.8082	0.0577	0.808
0.0409	0.8495	0.0409	0.8493	0.0409	0.8491	0.0409	0.8489
0.0574	0.9069	0.0574	0.9067	0.0575	0.9066	0.0576	0.9065
0.0262	0.9331	0.0263	0.933	0.0263	0.9329	0.0264	0.9329
0.0359	0.969	0.0359	0.9689	0.036	0.9689	0.036	0.9688
0.031	1	0.0311	1	0.0311	1	0.0312	1
G4		E6		G4		E4	
A4		E6		G4		E4	

(Figure 5: The table shows the steady state of the melody.)

4.2 Development of the melody and Trails

Since there were only notes that are of equal time in the matrix, the music had very boring rhythm. To improve this situation, quarter rests were added to the transition matrix. The quarter rests will be represented as 0. All the quarter rests were added in according to the sample music. The process used to determine the previous transition matrix was repeated to determine the developed transition matrix (work shown in Table 4 and the developed initial form shown in Table 5).

Developed Transition Matrix

	E4	F4	G4	A4	B4	C5	E5	F5	G5	A5	B5	C6	D6	E6	F6	G6	A6	0
E4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/15	0	0
F4	1	0	12/59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G4	0	1/13	0	1/5	0	23/31	0	0	1/19	7/54	0	1/24	0	0	0	0	0	1/9
A4	0	1/13	1/59	0	1/2	0	0	0	0	0	0	0	0	0	0	0	1/13	1/9
B4	0	0	0	1/5	0	1/62	0	0	0	0	0	0	0	0	0	0	0	1/9
C5	0	0	23/59	1/5	0	0	24/29	3/7	0	0	0	0	0	0	0	0	0	0
E5	0	0	10/59	0	0	6/31	0	1/7	0	1/54	0	1/24	0	0	0	0	0	0
F5	0	0	12/59	0	0	1/31	4/29	0	8/57	1/54	1	0	0	0	0	0	0	0
G5	0	3/13	0	0	0	0	0	9/28	4/57	14/27	0	13/24	0	0	0	0	0	4/9
A5	0	7/13	0	1/5	0	0	0	1/14	29/57	0	0	0	8/17	1/6	0	2/15	1/13	0
B5	0	0	0	0	0	0	0	0	0	1/54	0	0	0	0	0	0	0	0
C6	0	1/13	0	0	0	0	1/29	1/28	4/57	0	0	5/24	0	5/24	3/11	0	0	0
D6	0	0	0	0	0	1/62	0	0	0	4/27	0	0	8/17	0	0	0	0	1/9
E6	0	0	0	0	0	0	0	0	0	4/27	0	0	0	1/3	8/11	0	0	0
F6	0	0	0	0	0	0	0	0	0	0	0	0	0	7/24	0	4/15	0	0
G6	0	0	0	0	0	0	0	0	4/57	0	0	1/6	0	0	0	0	7/13	0
A6	0	0	0	0	0	0	0	0	1/57	0	0	0	0	0	0	8/15	4/13	0
0	0	0	1/59	1/5	1/2	0	0	0	4/57	0	0	0	1/17	0	0	0	0	1/9
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

(Table 7: The table shows the developed transition matrix of the Markov Chain matrix.)

	E4	F4	G4	A4	B4	C5	E5
E4	0	0	0	0	0	0	0
F4	1	0	0.203389830508475	0	0	0	0
G4	0	0.076923076923076	0	0.2	0	0.741935483870968	0
A4	0	0.076923076923076	0.016949152542373	0	0.5	0	0
B4	0	0	0	0.2	0	0.016129032258064	0
C5	0	0	0.389830508474576	0.2	0	0	0.827586206896552
E5	0	0	0.169491525423729	0	0	0.193548387096774	0
F5	0	0	0.203389830508475	0	0	0.032258064516129	0.137931034482759
G5	0	0.230769230769231	0	0	0	0	0
A5	0	0.538461538461538	0	0.2	0	0	0
B5	0	0	0	0	0	0	0
C6	0	0.076923076923076	0	0	0	0	0.034482758620689
D6	0	0	0	0	0	0.016129032258064	0
E6	0	0	0	0	0	0	0
F6	0	0	0	0	0	0	0
G6	0	0	0	0	0	0	0
A6	0	0	0	0	0	0	0
0	0	0	0.016949152542373	0.2	0.5	0	0
	=SUM(B2:B19)	=SUM(C2:C19)	=SUM(D2:D19)	=SUM(E2:E19)	=SUM(F2:F19)	=SUM(G2:G19)	=SUM(H2:H19)

(Table 8: The table shows part of the calculation for Table 7.)

4.3 Trail of simulated notes

Finally, a spreadsheet of 50 simulated notes using the developed transition matrix including the states and the process is created to provide evidence for further evaluation and investigation.

2		3		4		5		6		7		8		9		10	
0	0	0	0	0	0	0.000901	0.000901	0.00087	0.00087	0.00149	0.00149	0.00142	0.00142	0.00177	0.00177	0.00177	0.00177
0	0	0.040678	0.04068	0.039973	0.039973	0.022412	0.023313	0.03865	0.03952	0.02841	0.02989	0.0336	0.03502	0.03088	0.03265	0.03174	0.03351
0.2	0.2	0.196535	0.23721	0.110191	0.150164	0.185602	0.208915	0.1354	0.17492	0.15789	0.18778	0.14484	0.17986	0.14736	0.18001	0.14399	0.17751
0	0.2	0.125612	0.36283	0.033141	0.183305	0.033359	0.242273	0.01953	0.19445	0.01765	0.20543	0.01496	0.19482	0.01437	0.19438	0.01379	0.19129
0.2	0.4	0.025448	0.38827	0.040337	0.223642	0.017627	0.2599	0.01388	0.20833	0.01093	0.21636	0.00916	0.20398	0.00865	0.20303	0.0081	0.1994
0.2	0.6	0.077966	0.46624	0.186678	0.41032	0.131542	0.391442	0.16059	0.36892	0.14796	0.36432	0.1505	0.35448	0.14766	0.35069	0.14594	0.34534
0	0.6	0.076312	0.54255	0.055663	0.465983	0.071232	0.462674	0.06744	0.43636	0.07003	0.43435	0.06833	0.42281	0.06843	0.41912	0.06728	0.41262
0	0.6	0.050833	0.59338	0.083748	0.549731	0.052778	0.515452	0.08273	0.51909	0.06409	0.49844	0.07386	0.49667	0.06766	0.48677	0.06987	0.48249
0	0.6	0.192593	0.78598	0.095069	0.644801	0.182608	0.69806	0.12343	0.64252	0.15812	0.65656	0.13529	0.63196	0.14737	0.63415	0.1397	0.62219
0.2	0.8	0	0.78598	0.1795	0.8243	0.105568	0.803628	0.15074	0.79326	0.12617	0.78273	0.14012	0.77207	0.13076	0.7649	0.13604	0.75824
0	0.8	0.003704	0.78968	0	0.8243	0.003324	0.806952	0.00195	0.79522	0.00279	0.78552	0.00234	0.77441	0.00259	0.7675	0.00242	0.76066
0	0.8	0	0.78968	0.027264	0.851565	0.024751	0.831704	0.03334	0.82856	0.03444	0.81996	0.03839	0.8128	0.0393	0.8068	0.04132	0.80198
0	0.8	0.055078	0.84476	0.041133	0.892698	0.056948	0.888652	0.04964	0.8782	0.05271	0.87268	0.04912	0.86192	0.04954	0.85634	0.04791	0.8499
0	0.8	0.02963	0.87439	0.009877	0.902574	0.03617	0.924822	0.03241	0.91061	0.04334	0.91601	0.04433	0.90626	0.04887	0.9052	0.05022	0.90012
0	0.8	0	0.87439	0.008642	0.911216	0.006485	0.931306	0.01403	0.92464	0.01539	0.93141	0.01833	0.92458	0.02002	0.92522	0.02132	0.92144
0	0.8	0	0.87439	0.013515	0.924732	0.013035	0.944341	0.02228	0.94692	0.02133	0.95274	0.02659	0.95117	0.02651	0.95174	0.02907	0.95051
0	0.8	0	0.87439	0.003379	0.92811	0.009916	0.954257	0.01321	0.96012	0.01811	0.97085	0.01972	0.97089	0.02262	0.97436	0.02369	0.9742
0.2	1	0.125612	1	0.07189	1	0.045743	1	0.03988	1	0.02915	1	0.02911	1	0.02564	1	0.0258	1
A5		A4		G4		F5		G5		E6		G6		A5		G5	
B4		D6		F4		G5		A5		G4		F5		F5		G6	
11		12		13		14		15		16		17		18		19	
0.00194	0.00194	0.001963	0.00196	0.002046	0.002046	0.002073	0.002073	0.00212	0.00212	0.00214	0.00214	0.00216	0.00216	0.00218	0.00218	0.00219	0.00219
0.03105	0.03299	0.031034	0.033	0.030761	0.032807	0.030636	0.032709	0.03048	0.03259	0.03037	0.03251	0.03026	0.03242	0.03019	0.03236	0.03012	0.0323
0.14306	0.17605	0.14159	0.17459	0.140568	0.173375	0.139644	0.172353	0.13891	0.1715	0.13829	0.17079	0.13779	0.17021	0.13737	0.16973	0.13703	0.16934
0.01362	0.18967	0.013494	0.18808	0.013446	0.186821	0.013413	0.185766	0.0134	0.1849	0.01339	0.18418	0.01338	0.18359	0.01337	0.1831	0.01337	0.18271
0.00798	0.19765	0.007804	0.19588	0.00776	0.194581	0.007695	0.193461	0.00767	0.19257	0.00764	0.19182	0.00763	0.19122	0.00762	0.19072	0.00761	0.19032
0.14452	0.34217	0.142905	0.33879	0.141818	0.3364	0.140701	0.334163	0.13989	0.33246	0.13915	0.33097	0.13858	0.3298	0.13809	0.32881	0.1377	0.32802
0.06688	0.40904	0.066109	0.4049	0.06568	0.40208	0.065175	0.399338	0.06484	0.3973	0.0645	0.39547	0.06426	0.39406	0.06404	0.39285	0.06387	0.39189

(Figure 6: The screenshot of part of the trail.)

2		3		4	
=MMULT(B2:S19,U2:	=SUM($A$23:A23)	=MMULT(B2:S19,A23:A	=SUM($D$23:D23)	=MMULT(B2:S19,D23:D	=SUM($G$23:G23)
	=SUM($A$23:A24)		=SUM($D$23:D24)		=SUM($G$23:G24)
	=SUM($A$23:A25)		=SUM($D$23:D25)		=SUM($G$23:G25)
	=SUM($A$23:A26)		=SUM($D$23:D26)		=SUM($G$23:G26)
	=SUM($A$23:A27)		=SUM($D$23:D27)		=SUM($G$23:G27)
	=SUM($A$23:A28)		=SUM($D$23:D28)		=SUM($G$23:G28)
	=SUM($A$23:A29)		=SUM($D$23:D29)		=SUM($G$23:G29)
	=SUM($A$23:A30)		=SUM($D$23:D30)		=SUM($G$23:G30)
	=SUM($A$23:A31)		=SUM($D$23:D31)		=SUM($G$23:G31)
	=SUM($A$23:A32)		=SUM($D$23:D32)		=SUM($G$23:G32)
	=SUM($A$23:A33)		=SUM($D$23:D33)		=SUM($G$23:G33)
	=SUM($A$23:A34)		=SUM($D$23:D34)		=SUM($G$23:G34)
	=SUM($A$23:A35)		=SUM($D$23:D35)		=SUM($G$23:G35)
	=SUM($A$23:A36)		=SUM($D$23:D36)		=SUM($G$23:G36)
	=SUM($A$23:A37)		=SUM($D$23:D37)		=SUM($G$23:G37)
	=SUM($A$23:A38)		=SUM($D$23:D38)		=SUM($G$23:G38)
	=SUM($A$23:A39)		=SUM($D$23:D39)		=SUM($G$23:G39)
	=SUM($A$23:A40)		=SUM($D$23:D40)		=SUM($G$23:G40)
=INDEX(A$2:A$19,COUNTIF(B$23:B$40,"<		=INDEX(A$2:A$19,COUNTIF(E$23:E$40,"<="&		=INDEX(A$2:A$19,COUNTIF(H$23:H$40,"<="&R	
B4		D6		F4	
11		12		13	
=MMULT(B2:S19,Y23	=SUM($A$45:A45)	=MMULT(B2:S19,A45:A	=SUM($D$45:D45)	=MMULT(B2:S19,D45:D	=SUM($G$45:G45)
	=SUM($A$45:A46)		=SUM($D$45:D46)		=SUM($G$45:G46)
	=SUM($A$45:A47)		=SUM($D$45:D47)		=SUM($G$45:G47)
	=SUM($A$45:A48)		=SUM($D$45:D48)		=SUM($G$45:G48)
	=SUM($A$45:A49)		=SUM($D$45:D49)		=SUM($G$45:G49)
	=SUM($A$45:A50)		=SUM($D$45:D50)		=SUM($G$45:G50)
	=SUM($A$45:A51)		=SUM($D$45:D51)		=SUM($G$45:G51)

(Figure 7: The Screenshot shows the calculation formulas used in Figure 6.)

According to the spreadsheet, the probabilities of notes stay the same after the 25th note, so the new steady state is also State 25 (evidence shown in Figure 8).

26		27		28		29	
0.00223	0.00223	0.00223	0.00223	0.00223	0.00223	0.00223	0.00223
0.0299	0.03213	0.02989	0.03212	0.02987	0.03211	0.02986	0.0321
0.13599	0.16812	0.13591	0.16803	0.13585	0.16796	0.1358	0.16789
0.01336	0.18148	0.01336	0.18138	0.01335	0.18131	0.01335	0.18125
0.00758	0.18906	0.00758	0.18896	0.00757	0.18888	0.00757	0.18882
0.13648	0.32553	0.13638	0.32534	0.13631	0.32519	0.13625	0.32507
0.06333	0.38887	0.06329	0.38864	0.06326	0.38845	0.06323	0.3883
0.066	0.45486	0.06598	0.45461	0.06596	0.45441	0.06594	0.45424
0.14373	0.5986	0.14376	0.59837	0.14378	0.59819	0.1438	0.59804
0.13515	0.73375	0.13518	0.73355	0.1352	0.73339	0.13522	0.73326
0.0025	0.73625	0.0025	0.73605	0.0025	0.73589	0.0025	0.73576
0.04555	0.7818	0.04558	0.78163	0.04561	0.7815	0.04563	0.78139
0.04707	0.82886	0.04708	0.82871	0.04708	0.82858	0.04709	0.82848
0.05815	0.88701	0.0582	0.88691	0.05825	0.88683	0.05829	0.88676
0.02585	0.91286	0.02588	0.91279	0.02591	0.91273	0.02593	0.91269
0.03346	0.94631	0.03349	0.94628	0.03352	0.94625	0.03354	0.94623
0.02936	0.97568	0.0294	0.97568	0.02943	0.97568	0.02945	0.97568
0.02432	1	0.02432	1	0.02432	1	0.02432	1
A5		E6		G5		G5	
C5		C5		A5		G4	

(Figure 8: The screenshot shows the new steady point of the melody.)

5 Evaluation

5.1 Overview

During the process of generating a piece of melody, the melody has been generated using mathematical method and assisted by technology. Therefore, the result is reasonable. However,

since there still are many limitations, the developed music sounded different from the original music composed by an artist.

5.2 Strengths and limitations

The process had a lot of work applied to technology, which means most of the data was confirmed by technology. The mathematical concept of the Markov Chain matrix has also been applied during the process which met the goal. On the other hand, there are still a lot of limitations in the generated music which caused the generated music to sound different from the original music. Firstly, in Assumption 1, it was assumed that all notes are of equal time and there is not any sharps or flats, but in the original music there are various types of notes including whole notes, half notes, eighth notes an sixteenth notes as well as five sharps. Without this variety of notes, the music sounds boring. Secondly, the original music combines electric music and piano so there are many advanced chords in the original music, but not in the generated music. This is another reason why the generated music does not sound as good.

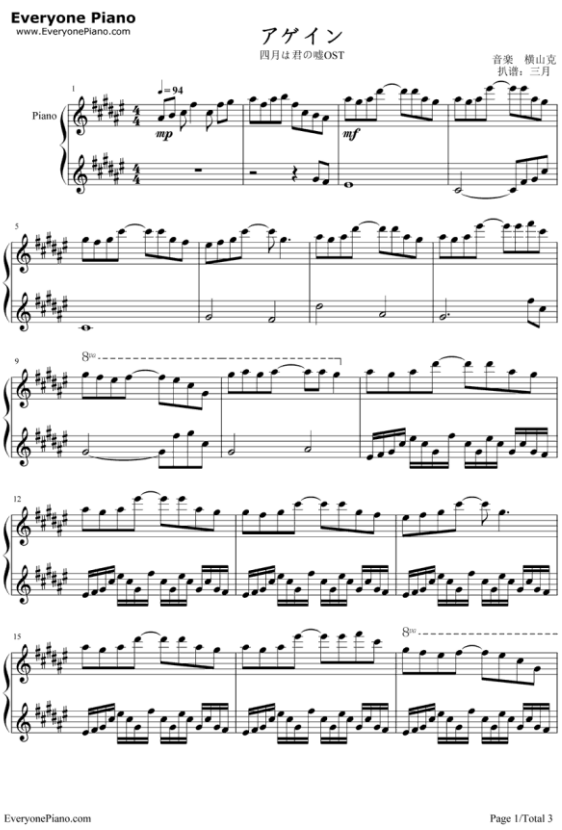
6 Conclusion

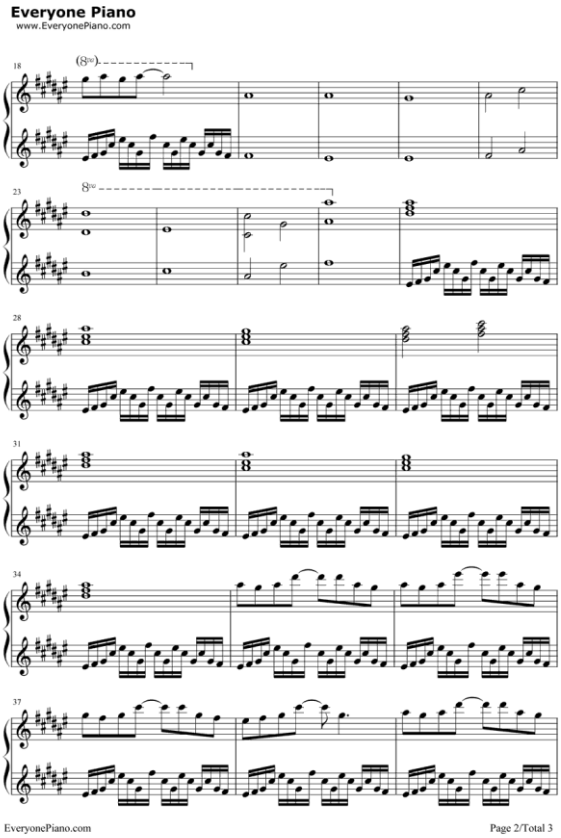
To conclude, a piece of melody was finally generated using mathematical method after a long and complex procedure of determination and refinement. 50 simulated notes were compared to the original music and the evaluation helped to conclude the report and further investigate on the solution achieved.

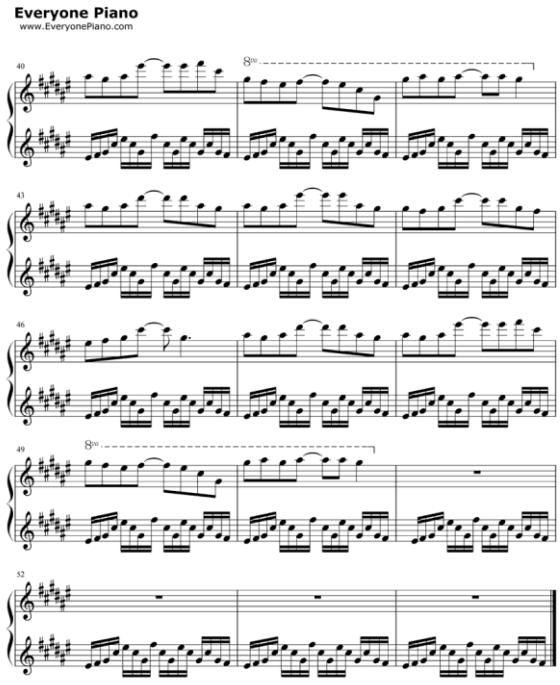
6.1 Recommendations – suggestions to make solution even more realistic and accurate

1. The matrix should include a variety type of notes, for example: C4 half, D5 quarter.
2. The matrix should Include chords in the matrix.

7 Appendixes







1		2		3		4		5	
0	0	0	0	0	0	0.000485164	0.00048516	0.000631234	0.00063
0	0	0.04067797	0.04068	0.04358909	0.043589089	0.042037045	0.04252221	0.041223368	0.04185
0.2	0.2	0.21431302	0.25499	0.20668214	0.250271225	0.200296172	0.24281838	0.191543637	0.2334
0.2	0.4	0.14338983	0.39838	0.05705237	0.307323593	0.03453063	0.27734901	0.021447609	0.25485
0.2	0.6	0.04322581	0.44161	0.03224822	0.339571816	0.014997719	0.29234673	0.010450379	0.2653
0.2	0.8	0.22135593	0.66296	0.22240924	0.561981056	0.21974366	0.51209039	0.205047747	0.47034
0	0.8	0.07631169	0.73927	0.08910264	0.651083691	0.094548963	0.60663935	0.091525567	0.56187
0	0.8	0.05083328	0.79011	0.0802547	0.731338395	0.07214418	0.67878353	0.081754971	0.64362
0	0.8	0.1037037	0.89381	0.05374469	0.785083081	0.121003502	0.79978703	0.100022914	0.74365
0.2	1	0.04	0.93381	0.12737362	0.912456703	0.083446547	0.88323358	0.119100003	0.86275
0	1	0.0037037	0.93751	0.00074074	0.913197443	0.002358771	0.88559235	0.001545306	0.86429
0	1	0	0.93751	0.02830373	0.941501175	0.028380559	0.91397291	0.040090422	0.90438
0	1	0.03285544	0.97037	0.02495756	0.966458739	0.034202148	0.94817506	0.032001833	0.93638
0	1	0.02962963	1	0.01580247	0.982261209	0.030422729	0.97859779	0.027266785	0.96365
0	1	0	1	0.00864198	0.990903184	0.006549708	0.9851475	0.01139823	0.97505
0	1	0	1	0.00727745	0.998180637	0.009468503	0.994616	0.016120645	0.99117
0	1	0	1	0.00181936	1	0.005384002	1	0.008829352	1
A4		B4		A4		F5		E5	
B4		G4		A4		A5		E5	
11		12		13		14		15	
0.00191	0.00191	0.00200923	0.00201	0.00207833	0.002078327	0.002142948	0.00214295	0.002190276	0.00219
0.03436	0.03627	0.03380064	0.03581	0.03334859	0.035426914	0.032974467	0.03511741	0.032672783	0.03486
0.15681	0.19308	0.15408519	0.1899	0.15190602	0.187332938	0.150105024	0.18522244	0.148649766	0.18351
0.01239	0.20547	0.01231065	0.20221	0.01225584	0.199588777	0.012215463	0.1974379	0.012184159	0.1957
0.00526	0.21073	0.00516698	0.20737	0.00510121	0.204689985	0.005046926	0.20248483	0.005005241	0.2007
0.1667	0.37743	0.16362286	0.371	0.16093703	0.36562701	0.158853217	0.36133804	0.157089174	0.35779
0.0772	0.45463	0.0759062	0.4469	0.07476848	0.440395489	0.073888281	0.43522633	0.073139896	0.43093
0.07196	0.52658	0.07091878	0.51782	0.0704242	0.510819694	0.069799391	0.50502572	0.069421706	0.50035
0.13125	0.65783	0.13325113	0.65107	0.13348608	0.644305778	0.134515771	0.63954149	0.134843418	0.6352
0.12507	0.7829	0.12521972	0.77629	0.12663364	0.77093942	0.126987067	0.76652855	0.127749504	0.76295
0.00226	0.78516	0.00231607	0.77861	0.00231888	0.773258304	0.002345067	0.76887362	0.002351612	0.7653
0.05194	0.8371	0.05282015	0.83143	0.05382325	0.82708155	0.054486568	0.82336019	0.055132043	0.82043
0.03917	0.87627	0.03965127	0.87108	0.03984957	0.866931119	0.040109036	0.86346923	0.040249888	0.86068
0.04783	0.9241	0.04977888	0.92086	0.05113394	0.918065063	0.052410384	0.91587961	0.05336365	0.91404
0.02105	0.94515	0.02198613	0.94284	0.02283215	0.940897208	0.023485857	0.93936547	0.024047467	0.93809
0.03014	0.97528	0.0311749	0.97402	0.03214421	0.973041421	0.032854144	0.97221961	0.033479469	0.97157
0.02472	1	0.02598124	1	0.02695858	1	0.027780389	1	0.028429948	1
G4		C5		G4		A6		G4	
A5		G5		c5		0		E5	
21		22		23		24		25	
0.00235	0.00235	0.00235709	0.00236	0.00236672	0.002366721	0.002374635	0.00237464	0.002381059	0.00238
0.03172	0.03407	0.03165013	0.03401	0.03159043	0.033957153	0.031541664	0.0339163	0.031501891	0.03388
0.14408	0.17815	0.14373058	0.17774	0.14344347	0.177400625	0.143209008	0.17712531	0.143017747	0.1769
0.01209	0.19024	0.01208123	0.18982	0.01207517	0.189475795	0.012070217	0.18919552	0.012066171	0.18897
0.00487	0.19511	0.00486406	0.19468	0.00485584	0.194331634	0.004849111	0.19404464	0.004843631	0.19381
0.15167	0.34679	0.15125471	0.34594	0.15091278	0.345244416	0.150634436	0.34467907	0.150406846	0.34422
0.07084	0.41763	0.07066698	0.4166	0.07052197	0.41576639	0.070403964	0.41508304	0.070307455	0.41452
0.06804	0.48567	0.06793457	0.48454	0.06785003	0.483616417	0.067779567	0.4828626	0.067722956	0.48225
0.13663	0.6223	0.13676625	0.62131	0.13686994	0.620486361	0.136960203	0.61982281	0.137030475	0.61928
0.1294	0.7517	0.12953014	0.75084	0.12964199	0.75012835	0.129728008	0.74955081	0.129801381	0.74908
0.00239	0.7541	0.00239636	0.75323	0.00239871	0.752527056	0.002400778	0.75195159	0.002402371	0.75148
0.05703	0.81112	0.05717271	0.8104	0.05729444	0.809821492	0.057392719	0.80934431	0.057473603	0.80896
0.04076	0.85188	0.04079852	0.8512	0.04082855	0.850650039	0.040853732	0.85019804	0.040873837	0.84983
0.05651	0.90839	0.05675497	0.90796	0.05695221	0.907602248	0.057114229	0.90731227	0.057245842	0.90708
0.02578	0.93417	0.02591082	0.93387	0.02602042	0.933622664	0.026109601	0.93342187	0.026182551	0.93326
0.03536	0.96953	0.03550081	0.96937	0.03561953	0.969242191	0.035715878	0.96913775	0.03579483	0.96905
0.03047	1	0.03063006	1	0.03075781	1	0.03086225	1	0.030947356	1
D6		E5		G4		D6		G5	
C5		A5		C5		E6		G4	

6		7		8		9		10	
0.00107	0.00107	0.00123	0.00123	0.00151	0.00151	0.00164	0.00164	0.00181	0.00181
0.03959	0.04066	0.03808	0.03932	0.03692	0.03843	0.03587	0.03751	0.03504	0.03685
0.18197	0.22263	0.17546	0.21477	0.16895	0.20738	0.16423	0.20174	0.16005	0.19691
0.01661	0.23924	0.01426	0.22903	0.01325	0.22063	0.01276	0.21451	0.01252	0.20943
0.0076	0.24684	0.00652	0.23555	0.00588	0.22651	0.00558	0.22009	0.00538	0.21481
0.19821	0.44505	0.18785	0.42339	0.18178	0.40829	0.17528	0.39537	0.17084	0.38565
0.08959	0.53464	0.08582	0.50921	0.08343	0.49172	0.08078	0.47615	0.07894	0.46458
0.07599	0.61063	0.07765	0.58686	0.07449	0.56621	0.07428	0.55043	0.07249	0.53708
0.12628	0.73691	0.1191	0.70596	0.12934	0.69555	0.12724	0.67767	0.13155	0.66863
0.10565	0.84256	0.12057	0.82653	0.11664	0.81218	0.12299	0.80066	0.1222	0.79083
0.00221	0.84476	0.00196	0.82848	0.00223	0.81442	0.00216	0.80282	0.00228	0.79311
0.04043	0.88519	0.04563	0.87412	0.04656	0.86098	0.04929	0.85211	0.05033	0.84344
0.03601	0.9212	0.0358	0.90991	0.03774	0.89871	0.03797	0.89008	0.03892	0.88236
0.03502	0.95622	0.03624	0.94615	0.04095	0.93966	0.04301	0.93308	0.04602	0.92837
0.01225	0.96848	0.01514	0.96128	0.01661	0.95627	0.01851	0.95159	0.01978	0.94815
0.01846	0.98693	0.02264	0.98392	0.02462	0.98089	0.02713	0.97872	0.0286	0.97675
0.01307	1	0.01608	1	0.01911	1	0.02128	1	0.02325	1
A5		G5		A5		G5		G4	
F4		A5		C5		0		A5	

16		17		18		19		20	
0.00223	0.00223	0.00226	0.00226	0.00229	0.00229	0.00231	0.00231	0.00233	0.00233
0.03242	0.03466	0.03222	0.03449	0.03206	0.03435	0.03192	0.03424	0.03181	0.03414
0.14745	0.18211	0.14648	0.18097	0.14569	0.18004	0.14504	0.17928	0.14451	0.17866
0.01216	0.19427	0.01214	0.19311	0.01212	0.19216	0.01211	0.19139	0.0121	0.19076
0.00497	0.19924	0.00494	0.19805	0.00492	0.19708	0.0049	0.19629	0.00489	0.19564
0.15569	0.35493	0.15452	0.35258	0.15359	0.35067	0.15281	0.3491	0.15219	0.34783
0.07255	0.42748	0.07205	0.42463	0.07166	0.42232	0.07133	0.42043	0.07106	0.41889
0.06903	0.49651	0.06876	0.49339	0.06851	0.49084	0.06833	0.48876	0.06817	0.48706
0.13542	0.63193	0.1357	0.62909	0.13605	0.62688	0.13626	0.62502	0.13648	0.62353
0.12808	0.76001	0.12852	0.75762	0.12878	0.75566	0.12905	0.75407	0.12923	0.75276
0.00237	0.76237	0.00237	0.75999	0.00238	0.75804	0.00238	0.75645	0.00239	0.75515
0.0556	0.81798	0.05602	0.81601	0.05635	0.81439	0.05662	0.81308	0.05684	0.812
0.0404	0.85838	0.0405	0.85651	0.04059	0.85498	0.04066	0.85373	0.04072	0.85271
0.0542	0.91258	0.05485	0.91136	0.05541	0.91039	0.05585	0.90958	0.05622	0.90893
0.02449	0.93707	0.02487	0.93623	0.02516	0.93555	0.02541	0.93499	0.02561	0.93454
0.03396	0.97103	0.03437	0.9706	0.03469	0.97024	0.03496	0.96995	0.03518	0.96972
0.02897	1	0.0294	1	0.02976	1	0.03005	1	0.03028	1
C5		A5		E5		A5		F6	
G5		E6		C5		G5		F5	

26		27		28		29		30	
0.00239	0.00239	0.00239	0.00239	0.00239	0.00239	0.0024	0.0024	0.0024	0.0024
0.03147	0.03386	0.03144	0.03383	0.03142	0.03382	0.0314	0.0338	0.03139	0.03379
0.14286	0.17672	0.14273	0.17657	0.14263	0.17645	0.14255	0.17635	0.14248	0.17626
0.01206	0.18878	0.01206	0.18863	0.01206	0.1885	0.01206	0.1884	0.01205	0.18832
0.00484	0.19362	0.00484	0.19346	0.00483	0.19334	0.00483	0.19323	0.00483	0.19315
0.15022	0.34384	0.15007	0.34353	0.14995	0.34328	0.14985	0.34308	0.14976	0.34291
0.07023	0.41407	0.07016	0.4137	0.07011	0.41339	0.07007	0.41315	0.07003	0.41294
0.06768	0.48175	0.06764	0.48134	0.06761	0.481	0.06758	0.48073	0.06756	0.48051
0.13709	0.61884	0.13714	0.61847	0.13718	0.61818	0.13721	0.61794	0.13723	0.61774
0.12986	0.74869	0.12991	0.74838	0.12995	0.74812	0.12998	0.74792	0.13	0.74775
0.0024	0.7511	0.0024	0.75079	0.00241	0.75053	0.00241	0.75032	0.00241	0.75015
0.05754	0.80864	0.05759	0.80838	0.05764	0.80817	0.05767	0.80799	0.0577	0.80785
0.04089	0.84953	0.0409	0.84928	0.04091	0.84908	0.04092	0.84892	0.04093	0.84879
0.05735	0.90688	0.05744	0.90672	0.05751	0.9066	0.05757	0.90649	0.05762	0.9064
0.02624	0.93312	0.02629	0.93301	0.02633	0.93293	0.02636	0.93285	0.02639	0.93279
0.03586	0.96898	0.03591	0.96893	0.03595	0.96888	0.03599	0.96884	0.03602	0.96881
0.03102	1	0.03107	1	0.03112	1	0.03116	1	0.03119	1
G4		E6		G4		E4		C5	
A4		0		0		E4		G5	

32			33			34			35			36	
0	0		0	0		0.000901018	0.000901018		0.000868994	0.000868994		0.001485269	0.001485269
0.040677966	0.040677966		0.03997327	0.03997327		0.022411657	0.023312675		0.03865058	0.039519574		0.028408176	0.029893446
0.196535245	0.237213211		0.110190649	0.150163919		0.185602013	0.208914688		0.13540098	0.174920554		0.1578868	0.187780245
0.125612053	0.362825264		0.033141089	0.183305009		0.033358554	0.242273243		0.019528526	0.194449079		0.017652798	0.205433043
0.025448029	0.388273292		0.040336823	0.223641832		0.017626883	0.259900125		0.013875928	0.208325007		0.010926833	0.216359876
0.077966102	0.466239394		0.186678037	0.410319868		0.131542094	0.391442219		0.160594725	0.368919733		0.147958732	0.364318608
0.076311686	0.54255108		0.055663169	0.465983037		0.07123173	0.462673949		0.067443758	0.43636349		0.070030915	0.434349523
0.050833283	0.593384363		0.083748299	0.549731336		0.052778343	0.515452292		0.082726163	0.519089654		0.064091855	0.498441378
0.192592593	0.785976956		0.095069341	0.644800677		0.182608142	0.698060434		0.12342713	0.642516784		0.1581176	0.656558978
0	0.785976956		0.179499754	0.824300431		0.105567807	0.803628242		0.150743431	0.793260215		0.126172735	0.782731713
0.003703704	0.789680659		0	0.824300431		0.00332407	0.806952311		0.001954959	0.795215174		0.002791545	0.785523258
0	0.789680659		0.027264095	0.851564526		0.024751365	0.831703676		0.033340262	0.828555436		0.034438446	0.819961703
0.055077658	0.844758318		0.041133311	0.892697837		0.056948073	0.888651749		0.049642986	0.878198422		0.052714893	0.872676596
0.02962963	0.874387947		0.009876543	0.90257438		0.03616981	0.924821559		0.032412446	0.910610868		0.043336876	0.916013472
0	0.874387947		0.008641975	0.911216356		0.00648473	0.93130629		0.014025504	0.924636372		0.015394707	0.931408179
0	0.874387947		0.01351527	0.924731625		0.013034912	0.944341202		0.022279037	0.94691541		0.021329498	0.952737676
0	0.874387947		0.003378817	0.928110443		0.009915663	0.954256865		0.013206578	0.960121987		0.018111104	0.97084878
0.125612053	1		0.071889557	1		0.045743135	1		0.039878013	1		0.02915122	1
A4			0			0			A6			A5	
D6			F4			G5			0			G4	
41			42			43			44			45	
0.001962957	0.001962957		0.002045894	0.002045894		0.002072887	0.002072887		0.002115994	0.002115994		0.002136883	0.002136883
0.031034457	0.032997414		0.030760838	0.032806732		0.030636075	0.032708963		0.030475124	0.032591118		0.030368433	0.032505316
0.141589583	0.174586996		0.140568392	0.173375124		0.139644329	0.172353292		0.138907827	0.171498944		0.138286317	0.170791632
0.013493536	0.188080532		0.013445806	0.186820929		0.013412896	0.185766187		0.013397233	0.184896178		0.013386058	0.18417769
0.007804432	0.195884964		0.007760334	0.194581264		0.007695248	0.193461435		0.007673797	0.192569975		0.007644535	0.191822225
0.142904627	0.338789591		0.141818279	0.336399542		0.140701321	0.334162756		0.1398936	0.332463575		0.13914932	0.330971544
0.066108688	0.404898279		0.065680354	0.402079896		0.065174784	0.399337541		0.064835025	0.397298601		0.064502525	0.39547407
0.068163653	0.473061932		0.067335701	0.469415597		0.06728318	0.466620721		0.066879847	0.464178448		0.066766815	0.462240885
0.141508407	0.614570338		0.143156211	0.612571808		0.14240862	0.609029342		0.143110314	0.607288762		0.142929962	0.605170847
0.13469747	0.749267809		0.133507401	0.746079209		0.134443734	0.743473075		0.134112249	0.741401011		0.134551557	0.739722405
0.002454327	0.751722135		0.002494398	0.748573607		0.002472359	0.745945434		0.002489699	0.74389071		0.00248356	0.742205965
0.042986335	0.794708471		0.043420538	0.791994144		0.043977587	0.789923021		0.044275709	0.78816642		0.044593131	0.786799095
0.047266456	0.841974927		0.047259846	0.83925399		0.047024888	0.83694791		0.047038168	0.835204587		0.046969178	0.833768273
0.053393293	0.89536822		0.054578923	0.893832914		0.05524939	0.8921973		0.055941619	0.891146206		0.056390874	0.890159146
0.02313572	0.91850394		0.023756619	0.917589533		0.024210402	0.916407702		0.024578381	0.915724587		0.024863836	0.915022982
0.030688408	0.949192349		0.031093311	0.948682844		0.031739907	0.948147609		0.032053238	0.947777824		0.032414779	0.947437761
0.025997227	0.975189576		0.026848902	0.975531745		0.027355812	0.975503421		0.02784352	0.975621345		0.028173004	0.975610765
0.024810424	1		0.024468255	1		0.024496579	1		0.024378655	1		0.024389235	1
E4			A5			A4			E5			A6	
E4			0			A5			D6			G5	

37			38			39	
0.001421967	0.001421967		0.001772523	0.001772523		0.001767419	0.001767419
0.033597839	0.035019805		0.030881621	0.032654144		0.031743525	0.033510944
0.144843301	0.179863106		0.147357426	0.18001157		0.143994333	0.177505277
0.014956895	0.19482		0.014368522	0.194380092		0.013788763	0.19129404
0.009156015	0.203976016		0.008652855	0.203032947		0.008104355	0.199398396
0.150504208	0.354480223		0.147657004	0.350689951		0.145942569	0.345340965
0.068325095	0.422805318		0.068425914	0.419115864		0.067279079	0.412620044
0.073864889	0.496670208		0.067657572	0.486773436		0.069872172	0.482492215
0.135285814	0.631956021		0.147371952	0.634145388		0.139701513	0.622193729
0.140117972	0.772073994		0.130756884	0.764902272		0.136044543	0.758238272
0.002336532	0.774410526		0.002594777	0.767497049		0.002421424	0.760659696
0.038386819	0.812797344		0.039304258	0.806801307		0.041322195	0.801981891
0.049124721	0.861922065		0.04953721	0.856338517		0.047913669	0.84989556
0.044334033	0.906256098		0.048865529	0.905204046		0.050220519	0.900116079
0.018327788	0.924583886		0.020020852	0.925224898		0.021322122	0.921438201
0.026587846	0.951171732		0.026511283	0.951736181		0.029073701	0.950511901
0.019722372	0.970894104		0.022622042	0.974358223		0.023685452	0.974197354
0.029105896	1		0.025641777	1		0.025802646	1
A5			0			E6	
F5			F5			G6	
46			47			48	
0.002160985	0.002160985		0.002175496	0.002175496		#NAME?	#NAME?
0.030262913	0.032423898		0.030185159	0.032360655		#NAME?	#NAME?
0.137785523	0.170209421		0.137369895	0.169730551		#NAME?	#NAME?
0.013379206	0.183588627		0.013373685	0.183104236		#NAME?	#NAME?
0.00763147	0.191220097		0.007616165	0.190720401		#NAME?	#NAME?
0.138581186	0.329801284		0.138088398	0.328808799		#NAME?	#NAME?
0.064258344	0.394059628		0.064039709	0.392848508		#NAME?	#NAME?
0.06654722	0.460606847		0.066447116	0.459295624		#NAME?	#NAME?
0.143260781	0.603867629		0.143256382	0.602552006		#NAME?	#NAME?
0.134508002	0.738375631		0.134734507	0.737286513		#NAME?	#NAME?
0.002491696	0.740867326		0.002490889	0.739777402		#NAME?	#NAME?
0.044794341	0.785661667		0.044985133	0.784762535		#NAME?	#NAME?
0.046990965	0.832652632		0.046980831	0.831743366		#NAME?	#NAME?
0.056813311	0.889465943		0.057113085	0.888856451		#NAME?	#NAME?
0.025091279	0.914557222		0.025272533	0.914128984		#NAME?	#NAME?
0.032632441	0.947189663		0.032845903	0.946974887		#NAME?	#NAME?
0.028464042	0.975653705		0.028675482	0.975650369		#NAME?	#NAME?
0.024346295	1		0.024349631	1		#NAME?	#NAME?
F5			C6			E4	
G5			F4			A6	

8 Reference list